CORRELATIONS

Electron-ion correlations (due to Coulomb)

$$E_F(r) = \frac{k_F^2(r)}{2m} + U(r)$$

$$k_F^3(r) = \left(2m\left(E_F(r) - U(r)\right)\right)^{3/2}$$

$$n(r) = \frac{k_F^3(r)}{3\pi^2}$$

$$\rho(r) = \rho_0(r)\left(1 - \frac{U(r)}{E_F}\right)^{3/2}$$ Only leading order position dependence

$$\nabla^2 U = -4\pi e\rho(r)$$

$$= -4\pi e\rho_0(r)\left(1 - \frac{U(r)}{E_F}\right)^{3/2}$$

$$= -4\pi e\rho_0(r) + \frac{6\pi e\rho_0}{E_F}U(r)$$

Transform to Fourier space

$$-q^2U(q) = -4\pi e\,\rho_{ext}(q) + q_{TF}^2 U(q) \qquad q_{TF}^2 = \frac{6\pi e\rho_0}{E_F}$$

$$U(q) = -\frac{4\pi e\,\rho_{ext}(q)}{q^2 + q_{TF}^2}$$

Note that this would give

from previous lecture

$$\varepsilon_r(q) = \frac{\rho_{ext}}{\rho_{ext} + \rho_{ocr}} = \frac{q^2 + q_{TF}^2}{q^2} = 1 - \frac{4\pi e^2}{q^2}\Pi$$

$$= 1 + \frac{q_{TF}^2}{q^2} = 1 + \frac{4\pi e^2}{q^2}\nu$$

Case study : point charge

$\rho_{ext}(\underline{r}) = Q.\delta(\underline{r})$
$\rho_{ext}(\underline{q}) = Q$

$$U(q) = -\frac{4\pi e Q}{q^2 + q_{TF}^2}$$

Optional

$$U(\underline{r}) = \int \frac{1}{(2\pi)^3} 4\pi q^2 . \, e^{i\underline{q}\cdot\underline{r}} \cdot \frac{-4\pi e Q}{q^2 + q_{TF}^2}$$
$$= -\frac{4\pi e Q}{(2\pi)^3} 2\pi \int_0^\infty dq \, . q^2 \int_{-1}^{1} dx \, e^{iqrx} \frac{1}{q^2 + q_{TF}^2}$$
$$= -\frac{eQ}{\pi} \int_0^\infty dq \frac{q^2}{q^2 + q_{TF}^2} \frac{2\sin qr}{qr}$$
$$= -\frac{eQ}{\pi r} \int_{-\infty}^{\infty} dq \frac{q \sin qr}{q^2 + q_{TF}^2} \qquad \text{poles at } q = \pm i q_{TF}$$

$$U(r) = -\frac{eQ}{r} e^{-q_{TF} r}$$

Electron - electron correlation (due to Pauli)

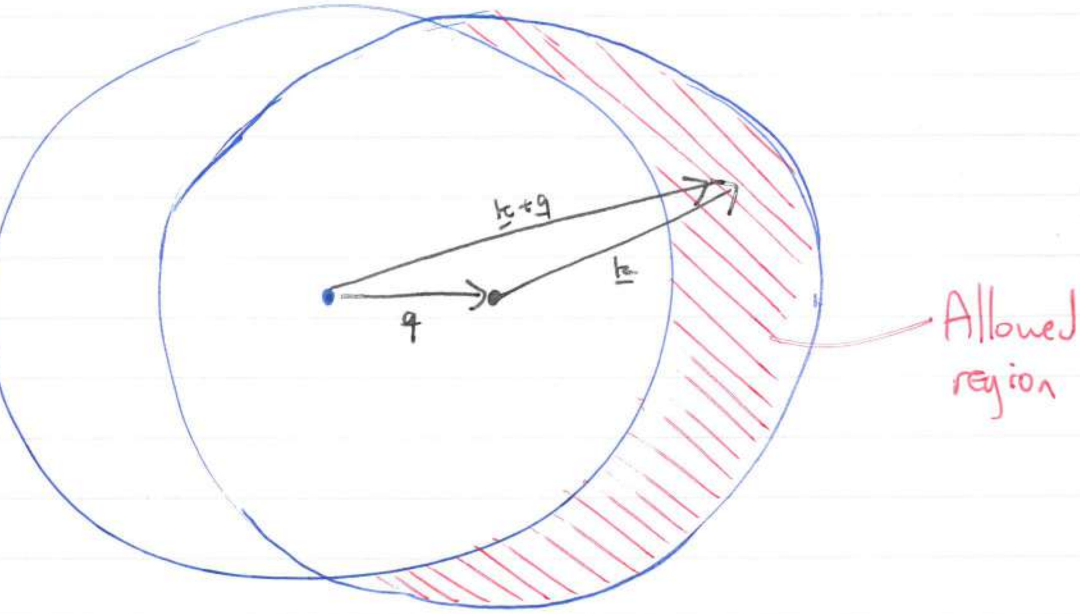
$$S_{\sigma\sigma'}(\underline{r}\,\underline{r}') = \langle n_\sigma(\underline{r})\, n_{\sigma'}(\underline{r}') \rangle \longrightarrow S_{\uparrow\downarrow} = 1.$$

$$S_{\sigma\sigma}(q) = \frac{1}{N}\langle \hat{n}_\sigma(q)\, \hat{n}_\sigma(-q) \rangle$$

Cannot naively evaluate due to quantum correlation physics

$$\begin{aligned}
S(q) &= \frac{1}{N}\sum_{kk'\sigma} \langle C^{+}_{k'} C_{k'+q} C^{+}_{k} C_{k-q} \rangle \quad \underline{k} \to \underline{k} + \underline{q} \\
&= \frac{1}{N}\sum_{kk'\sigma} \langle C^{+}_{k'} C_{k'+q} C^{+}_{k+q} C_{k} \rangle \\
&= \frac{1}{N}\sum_{kk'\sigma} \langle C_{k'+q} C^{+}_{k+q} C^{+}_{k'} C_{k} \rangle (-1)^2 \\
&= \frac{1}{N}\sum_{k\sigma} \langle C_{k+q} C^{+}_{k+q} \rangle \langle C^{+}_{k} C_{k} \rangle \\
&= \frac{2}{N}\sum_{k} \left(1 - f(\varepsilon_{k+q})\right) f(\varepsilon_k)
\end{aligned}$$

only get contributions at $\underline{k} = \underline{k}'$

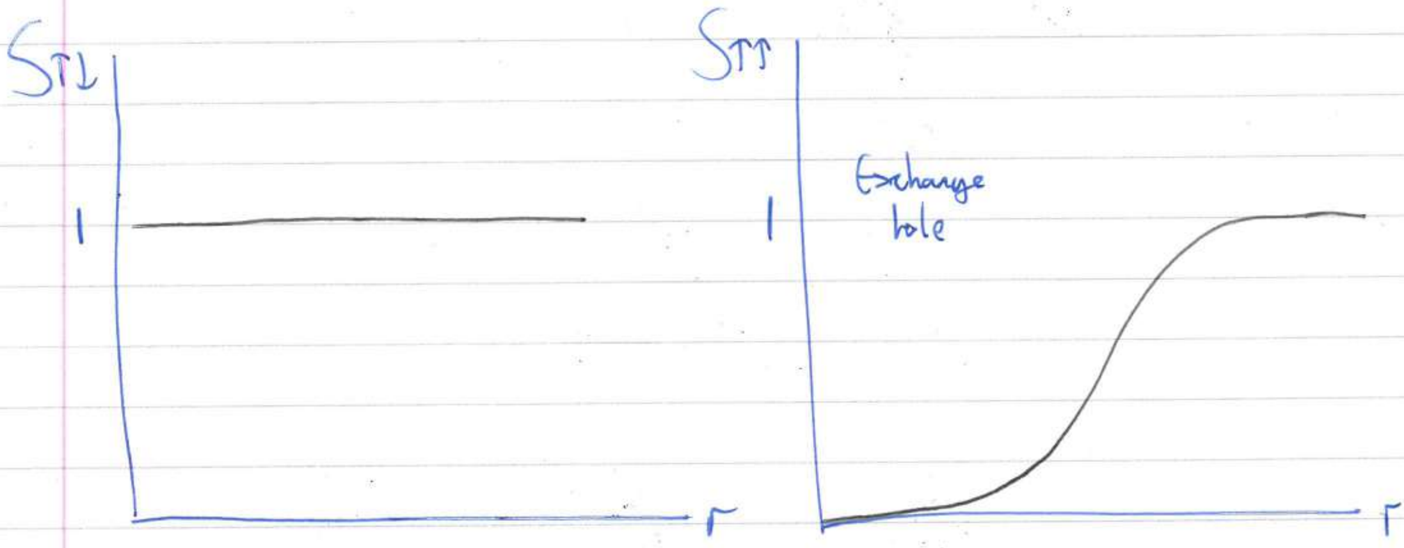


so as

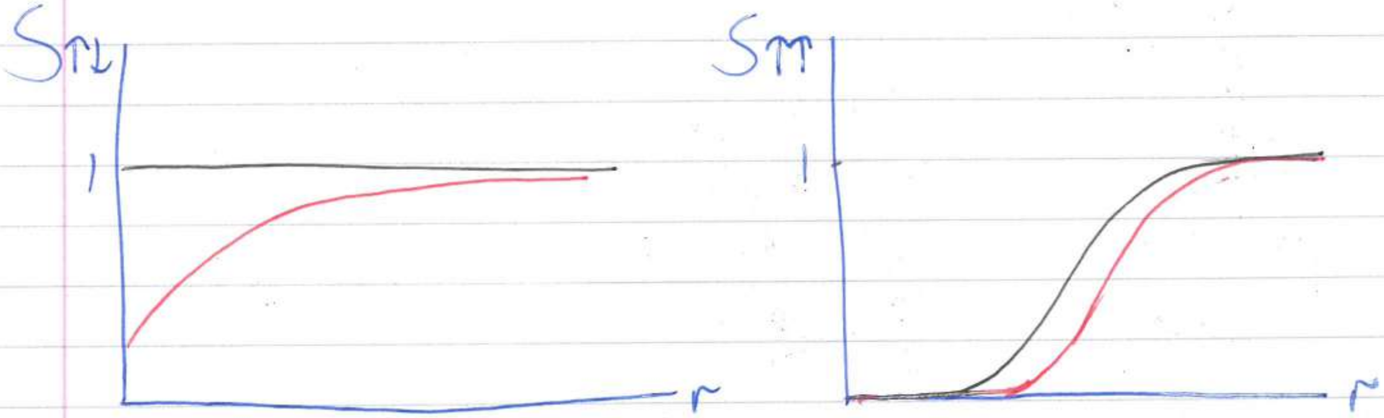
$q \to 0$	$S(q) \to 0$	phase space always less
$q > 2k_F$	$S(q) \to 1$	
$0 < q < 2k_F$	$S(q) = \frac{3}{5}\frac{\lvert q\rvert}{k_F} - \frac{1}{16}\left(\frac{q}{k_F}\right)^2$	

$$S_{\uparrow\downarrow} = 1$$

$$S_{\uparrow\uparrow} = 1 - 9\left(\frac{\sin k_F r - k_F r \cos k_F r}{(k_F r)^3}\right)^2$$



Ignore e-e interactions, in reality will push electrons apart so that



One body density matrix:

$$\left\langle c^{?}_{\alpha}(\underline{r})\, c^{?}_{\beta}(\underline{r}) \right\rangle$$

Examples:

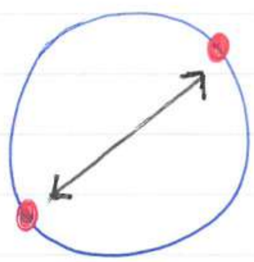
$$\left\langle c^{+}_{\uparrow}(\underline{r})\, c_{\uparrow}(\underline{r}) + c^{+}_{\downarrow}(\underline{r})\, c_{\downarrow}(\underline{r}) \right\rangle = \rho(r) \qquad \text{Density}$$

$$\frac{1}{2}\left\langle c^{+}_{\uparrow}(\underline{r})\, c_{\uparrow}(\underline{r}) - c^{+}_{\downarrow}(\underline{r})\, c_{\downarrow}(\underline{r}) \right\rangle = S_z(\underline{r}) \qquad \text{Magnetism}$$

or in general as spin is a vector

$$\frac{1}{2}\left\langle \begin{pmatrix} c^{+}_{\uparrow}(\underline{r}) & c^{+}_{\downarrow}(\underline{r}) \end{pmatrix} \begin{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{pmatrix} \begin{pmatrix} c_{\uparrow}(\underline{r}) \\ c_{\downarrow}(\underline{r}) \end{pmatrix} \right\rangle = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \underline{S}(\underline{r})$$

$$g \sum_{\underline{k}} \left\langle c_{\uparrow}(\underline{k})\, c_{\downarrow}(-\underline{k}) \right\rangle = \Delta \qquad \text{Superconducting order}$$



order between electrons sourced from opposing sides of Fermi surface

Useful to reveal phase transitions eg magnetism / superconductivity and response to external stimulae eg magnetic domains / flux lines

Single body do not reveal full consequences of e-e correlations as they do not contain relative electron position. To do this use two-body operators

$$\langle c_\alpha^+ c_\beta^+ c_\gamma c_\delta \rangle$$

Examples:

$\langle n(\underline{r}) n(\underline{r}') \rangle$ Density - density

$\langle \underline{S}(\underline{r}) \cdot \underline{S}(\underline{r}') \rangle$ Magnetic correlations

$\langle \Delta^*(\underline{r}) \cdot \Delta(\underline{r}') \rangle$ Superconducting

Time dependence → fluctuations

Fluctuations length scales diverge at second order transitions eg clouds have length scale of light

Spatial decay reveals properties about system.

Condensed matter physics is many-body + interactions
Laid foundations to explore condensed matter physics:
Second quantization to do many-body + quantum in commutators
How to write down Hamiltonian
How to probe emergent correlations
Future lectures: applications to specialist problems
further methods of analytical, computational, and experimental analysis.