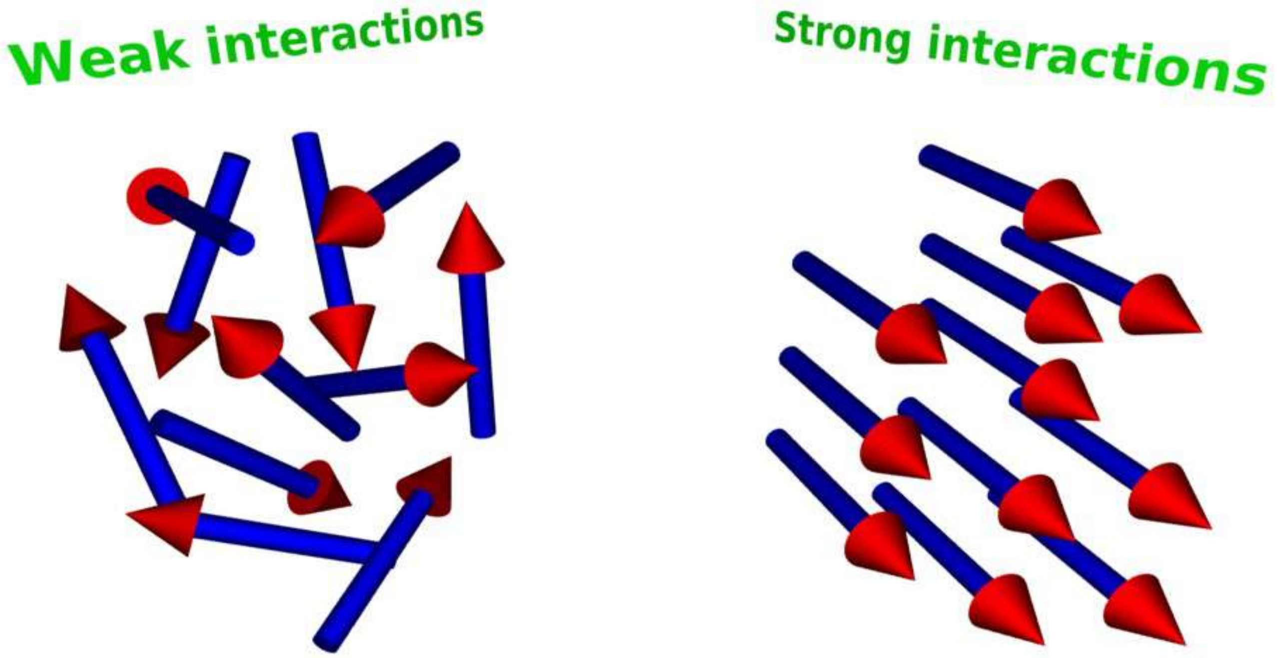
A repulsive atomic gas on the border of itinerant ferromagnetism



Gareth Conduit[1,2] , Ben Simons[3] & Ehud Altman[1]

1. Weizmann Institute, 2. Ben Gurion University, 3. University of Cambridge

G.J. Conduit & B.D. Simons, Phys. Rev. A **79**, 053606 (2009)

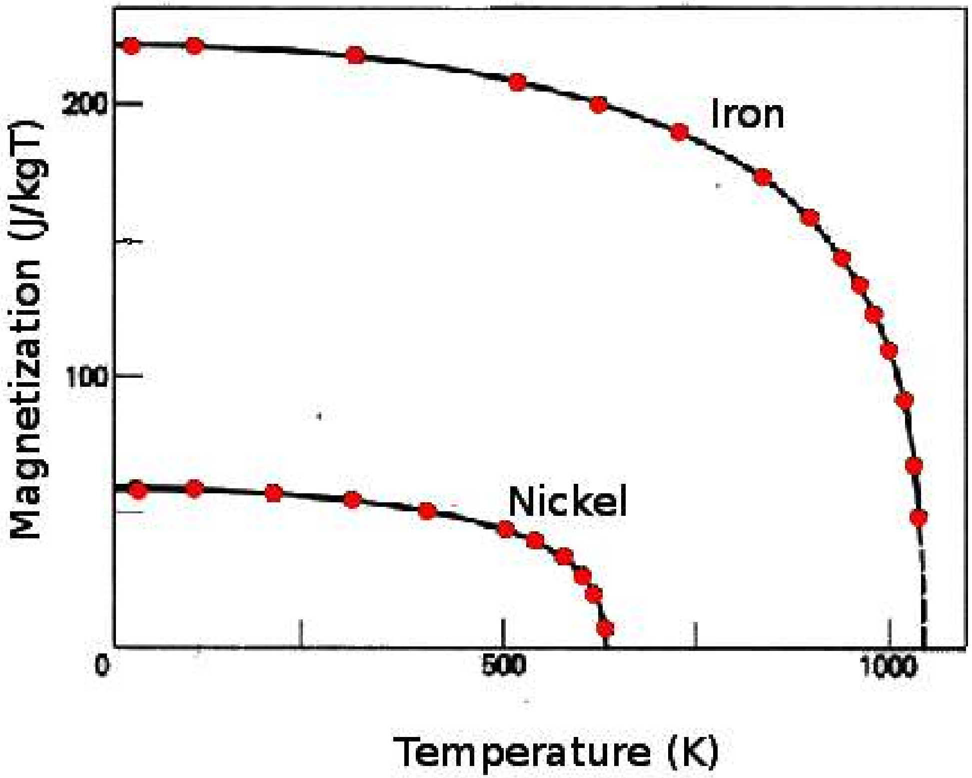
G.J. Conduit, A.G. Green & B.D. Simons, Phys. Rev. Lett. **103**, 207201 (2009)

G.J. Conduit & B.D. Simons, Phys. Rev. Lett. **103**, 200403 (2009)

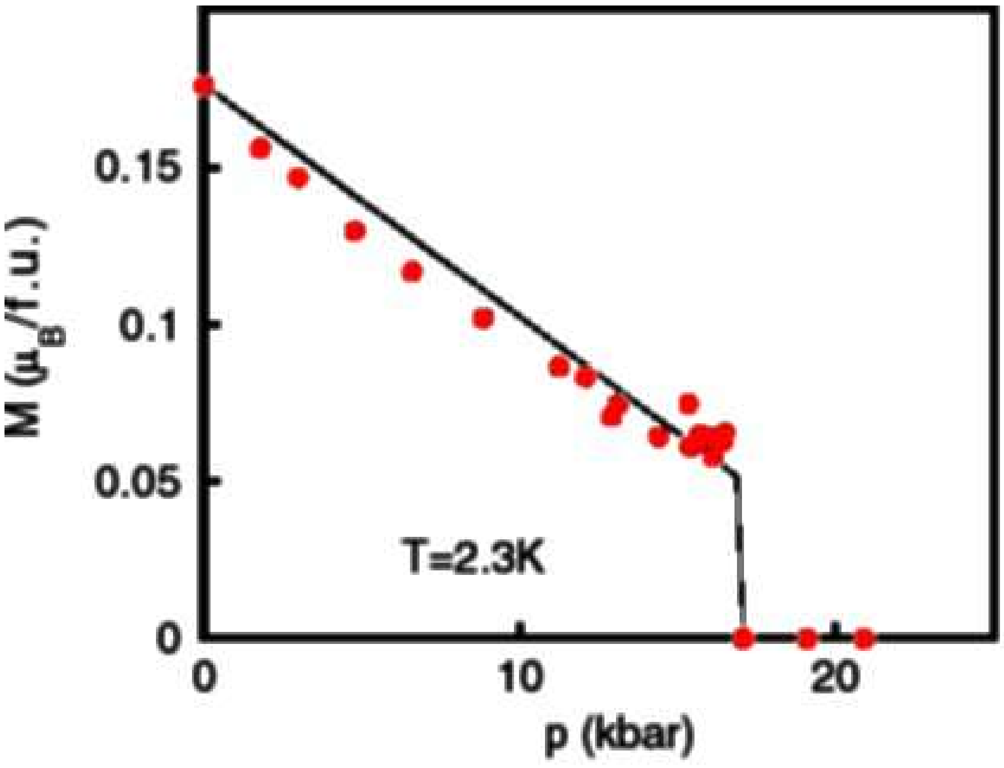
G.J Conduit & E. Altman, arXiv: 0911.2839

Ferromagnetism in solid state

Second order in iron & nickel

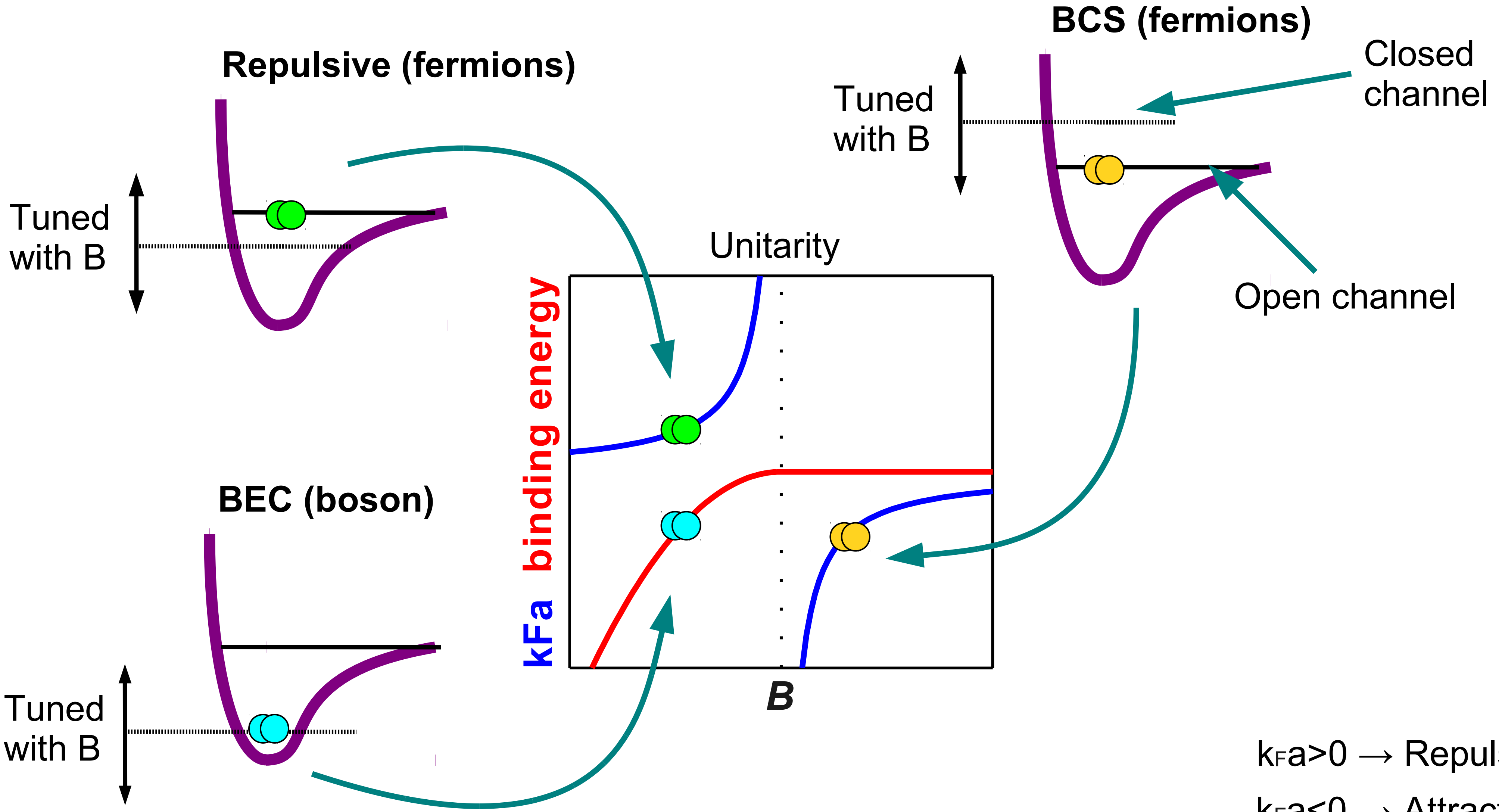


First order in $ZrZn_2$



Uhlarz *et al.*, PRL 2004

Feshbach resonance



$k_F a > 0 \rightarrow$ Repulsive

$k_F a < 0 \rightarrow$ Attractive

- Note instability to BEC molecular state on repulsive side of resonance

Stoner instability with repulsive interactions

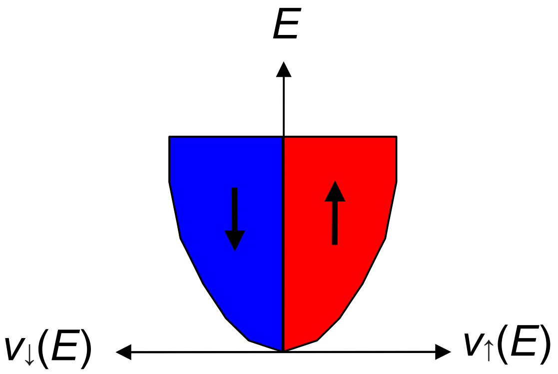
- Use two ^{6}Li states to represent pseudo up and down-spin electrons

$$\hat{H}=\sum_{\boldsymbol{k}\sigma}\epsilon_{\boldsymbol{k}}c^{\dagger}_{\boldsymbol{k}\sigma}c_{\boldsymbol{k}\sigma}+g\sum_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}}c^{\dagger}_{\boldsymbol{k}\uparrow}c^{\dagger}_{\boldsymbol{k}'+\boldsymbol{q}\downarrow}c_{\boldsymbol{k}'+\boldsymbol{q}\downarrow}c_{\boldsymbol{k}'\uparrow}$$

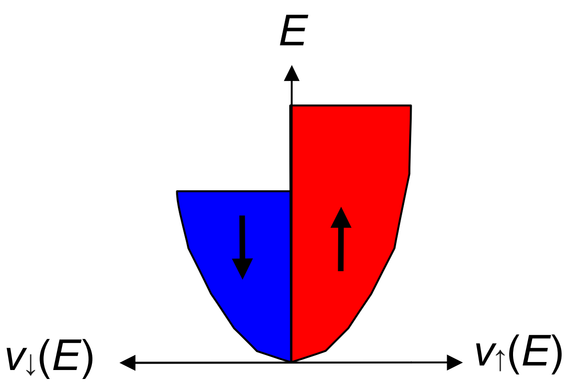
$$F=F_0+\frac{1-g\nu}{2\nu}m^2+um^4$$

- A Fermi surface shift increases the kinetic energy and potential energy falls
- Ferromagnetic transition occurs if $g\nu>1$

Not magnetised



Partially magnetised

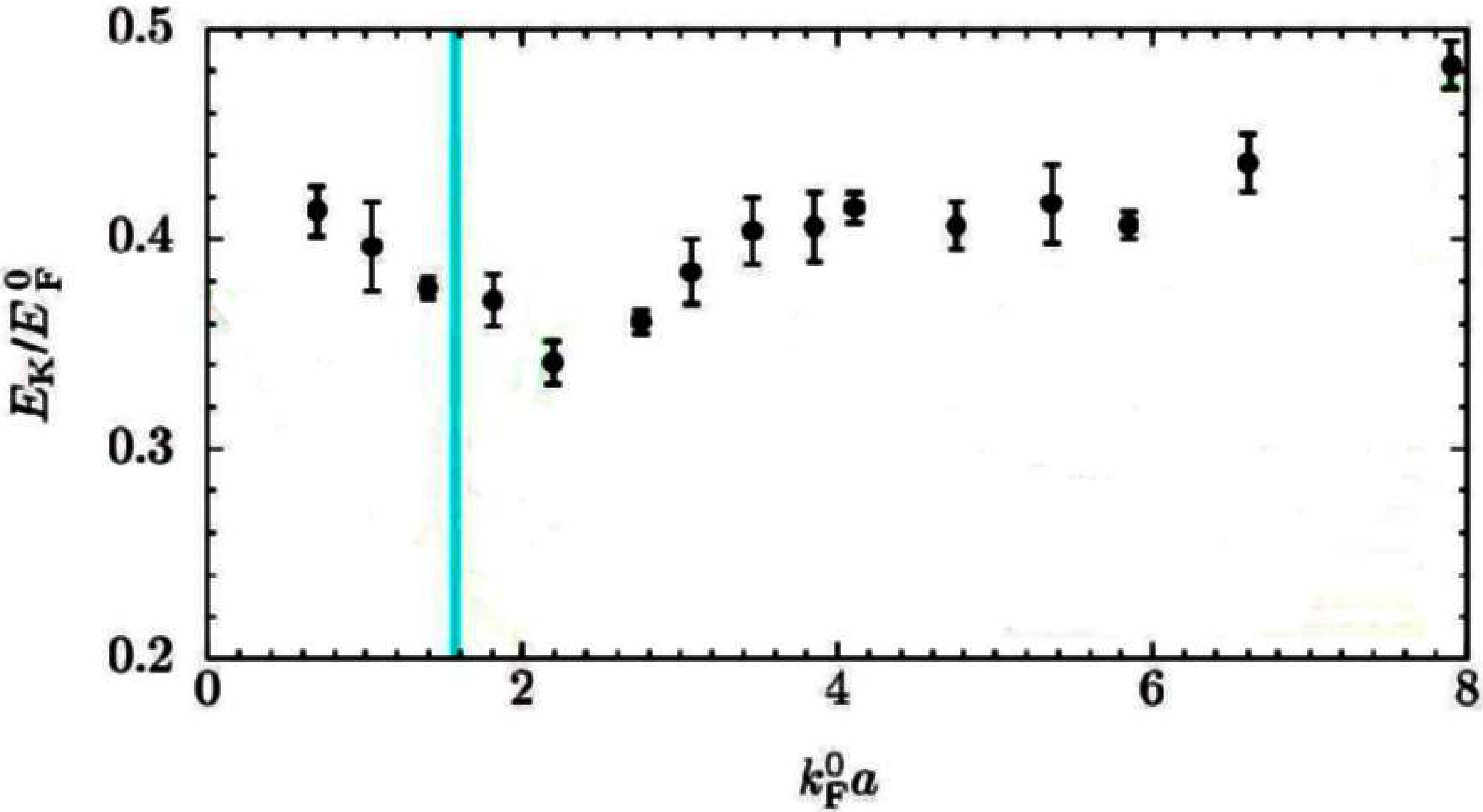


Conduit & Simons, Phys. Rev. A **79**, 053606 (2009)

Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

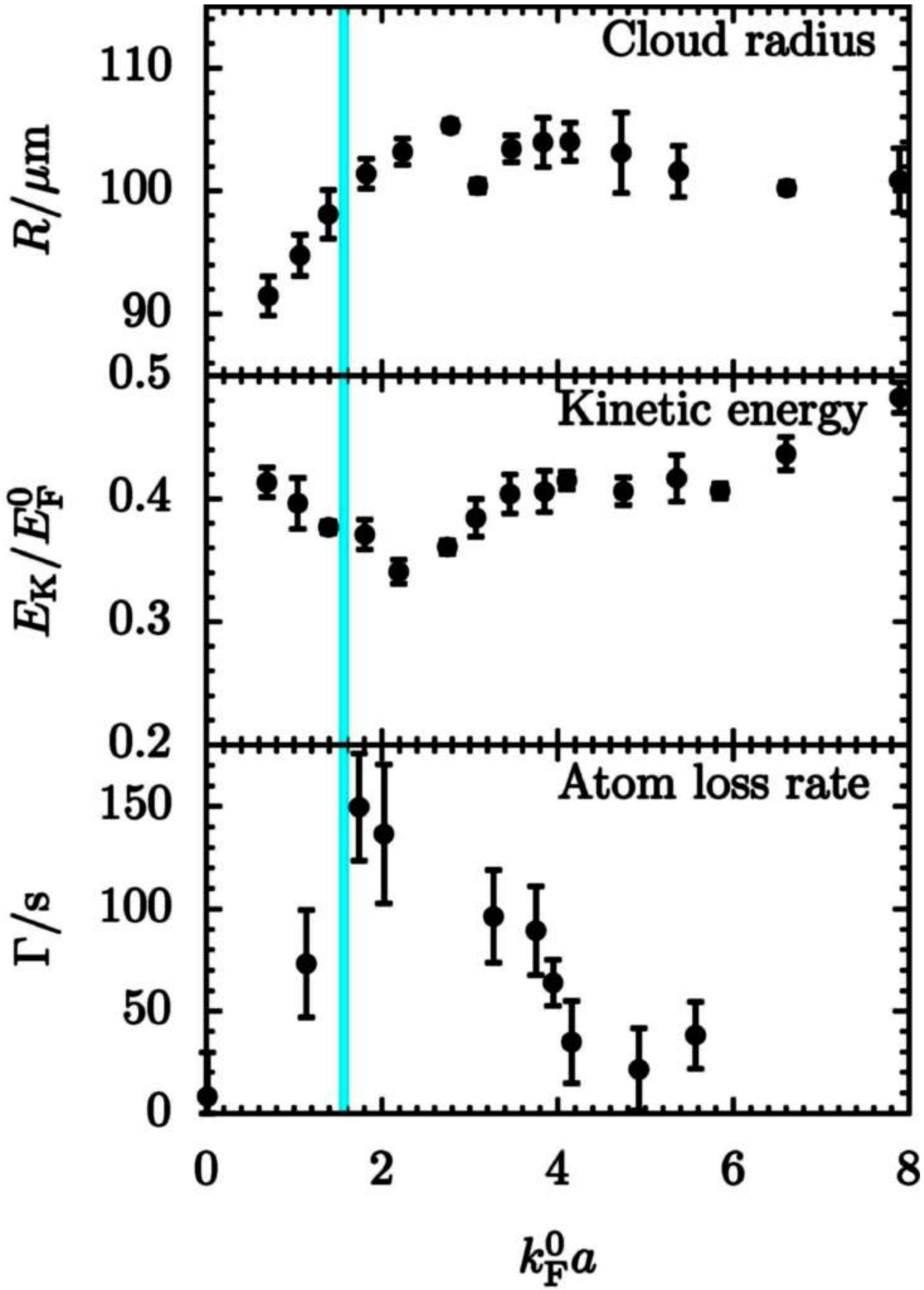
Experimental evidence for ferromagnetism

- Rise in kinetic energy also seen in experiment, $k_F a = \pi v g/2$



Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Further key experimental signatures

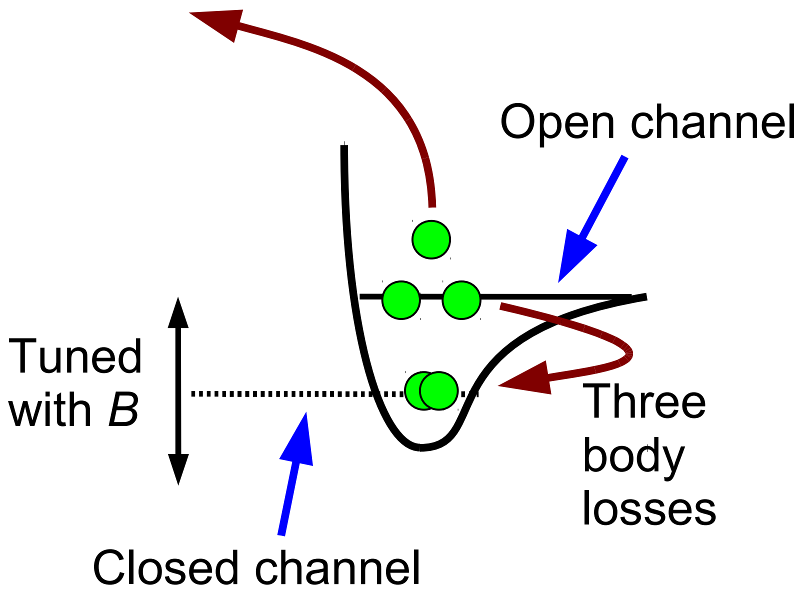


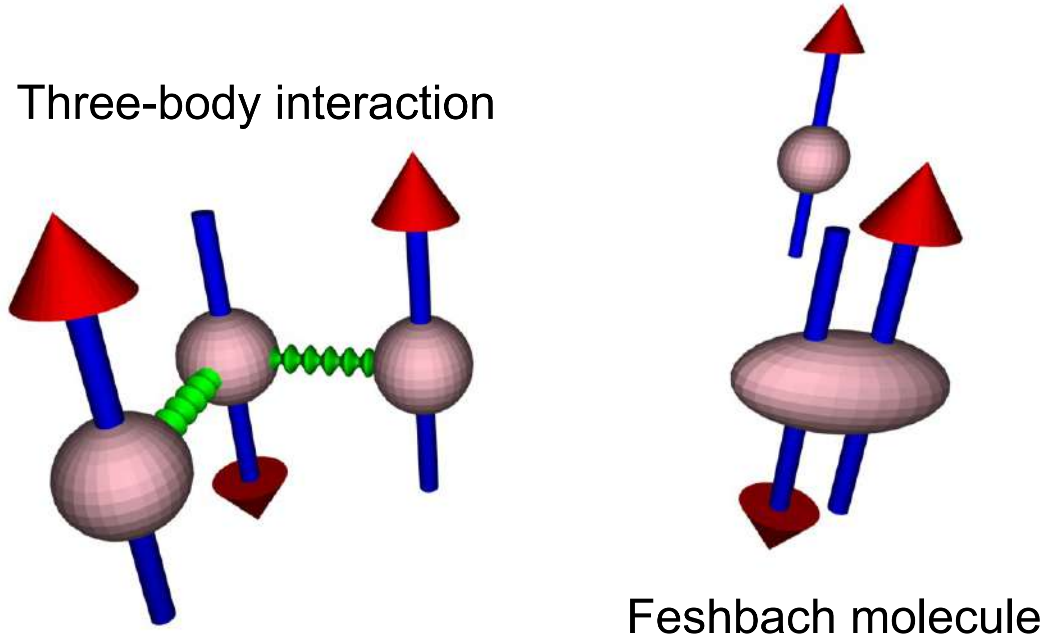
$$E_K \propto n^{5/3}$$

$$\Gamma \propto (k_F a)^6 n_\uparrow n_\downarrow (n_\uparrow + n_\downarrow)$$

Jo, Lee, Choi, Christensen, Kim, Thywissen, Pritchard & Ketterle, Science **325**, 1521 (2009)

Three body losses





- In boson systems, three-body scattering drives the formation of a Tonks-Girardeau gas [Syassen *et al*., Science **320**, 1329 (2009)]
- The experiment is an opportunity to study not only ferromagnetism but also three-body loss and dynamics

Outline

- Equilibrium analysis with mean field & fluctuation corrections
  - Fluctuation corrections lead to emergence of first order transition
- The Stoner transition in the presence of atom loss
  - Renormalization of interaction strength
  - Second order rather than first order transition
  - Modified collective modes

[1]Berdnikov *et al.*, PRB **79**, 224403 (2009), LeBlanc *et al.*, PRA **80**, 013607 (2009)
[2]Duine & MacDonald, PRL **95**, 230403 (2005)
[3]Babadi *et al.*, arXiv:0908.3483
[4]Zhai, PRA **80**, 051605(R) (2009)

Equilibrium study of ferromagnetism

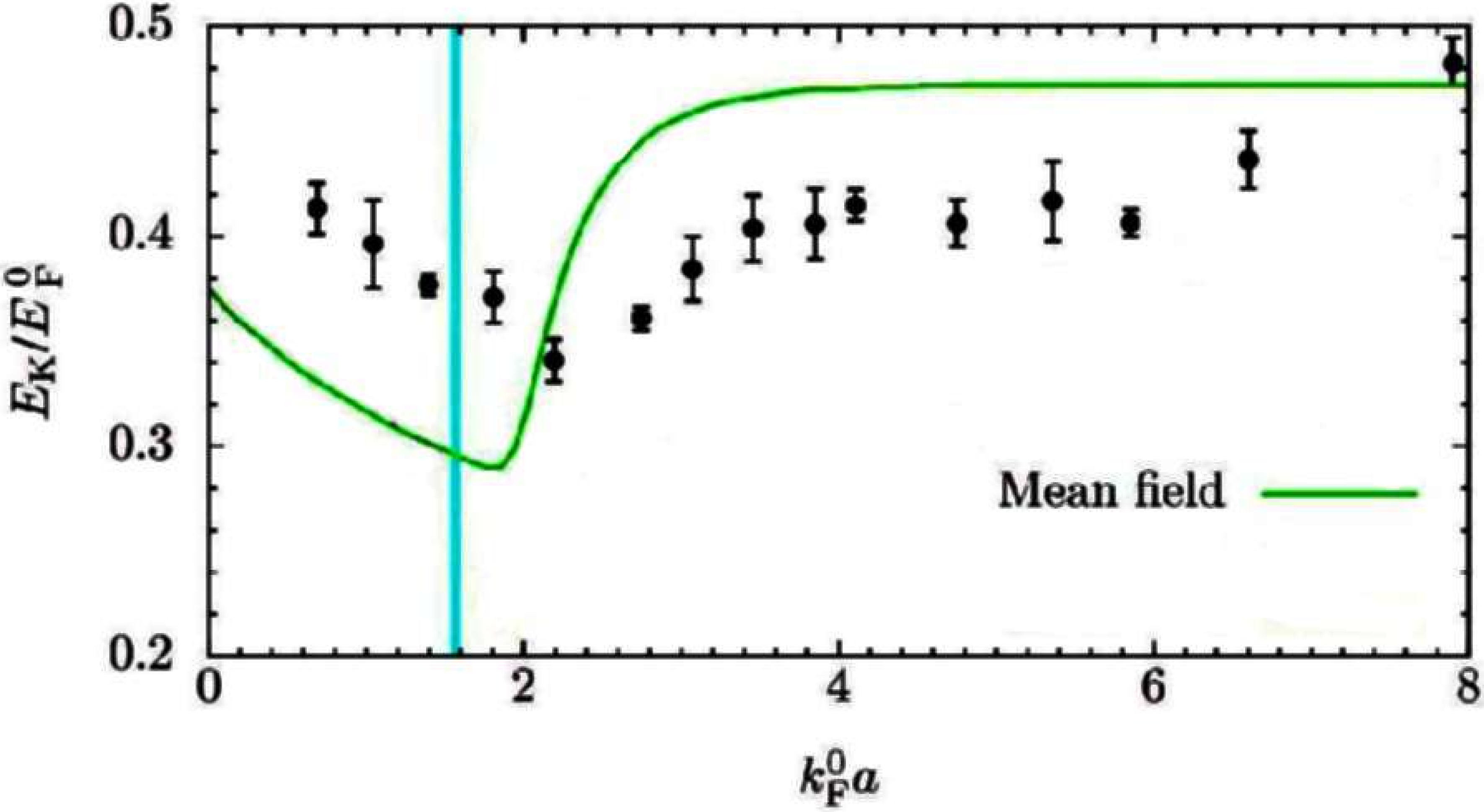
$$Z=\int D\psi \exp\left(-\iint d\tau\, d\boldsymbol{r} \sum_{\sigma} \bar{\psi}_{\sigma}(-i\hat{\omega}+\hat{\epsilon}-\mu)\psi_{\sigma} - g\,\bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Decouple with the average magnetisation *m* gives the Stoner criterion

$$F=F_0+\frac{1-g\nu}{2\nu}m^2+um^4+vm^6$$

Mean-field analysis & consequences of trap

- Recovers qualitative behavior[1] but transition at $k_F a$=1.8 instead of $k_F a$=2.2



[1]LeBlanc, Thywissen, Burkov & Paramekanti, Phys. Rev. A **80**, 013607 (2009) & Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Fluctuation corrections

$$Z=\int D\psi \exp\left(-\iint d\tau\, d\mathbf{r} \sum_{\sigma} \bar{\psi}_{\sigma}(-i\hat{\omega}+\hat{\epsilon}-\mu)\psi_{\sigma} - g\,\bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Integrate over fluctuations in magnetization and density and develop a perturbation theory in interaction strength

$$F=F_0+\frac{1-g\nu}{2\nu}m^2+um^4+vm^6+g^2\left(r\,m^2+w\,m^4\ln|m|\right) \qquad k_{\mathrm{F}}a_{\mathrm{crit}}=1.05$$

- First order transition[1]

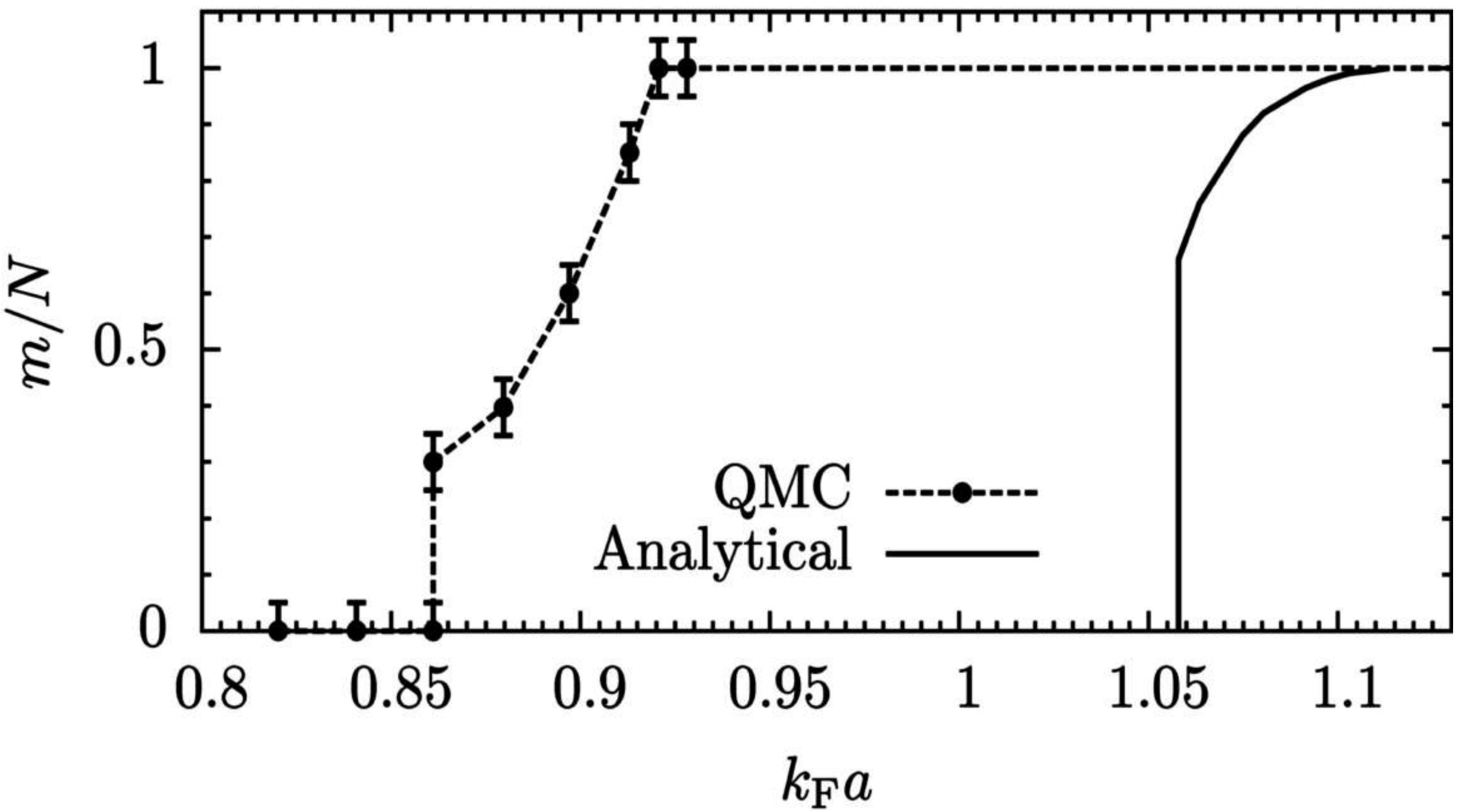
[1]Abrikosov (1958), Duine & MacDonald (2005), Belitz *et al*. Z. Phys. B (1997) & Conduit & Simons (2009)

Quantum Monte Carlo

$$F = F_0 + \frac{1 - g\nu}{2\nu} m^2 + u m^4 + v m^6 + g^2 \left(r\, m^2 + w\, m^4 \ln|m| \right)$$

$$k_F a_{\mathrm{crit}} = 1.05$$

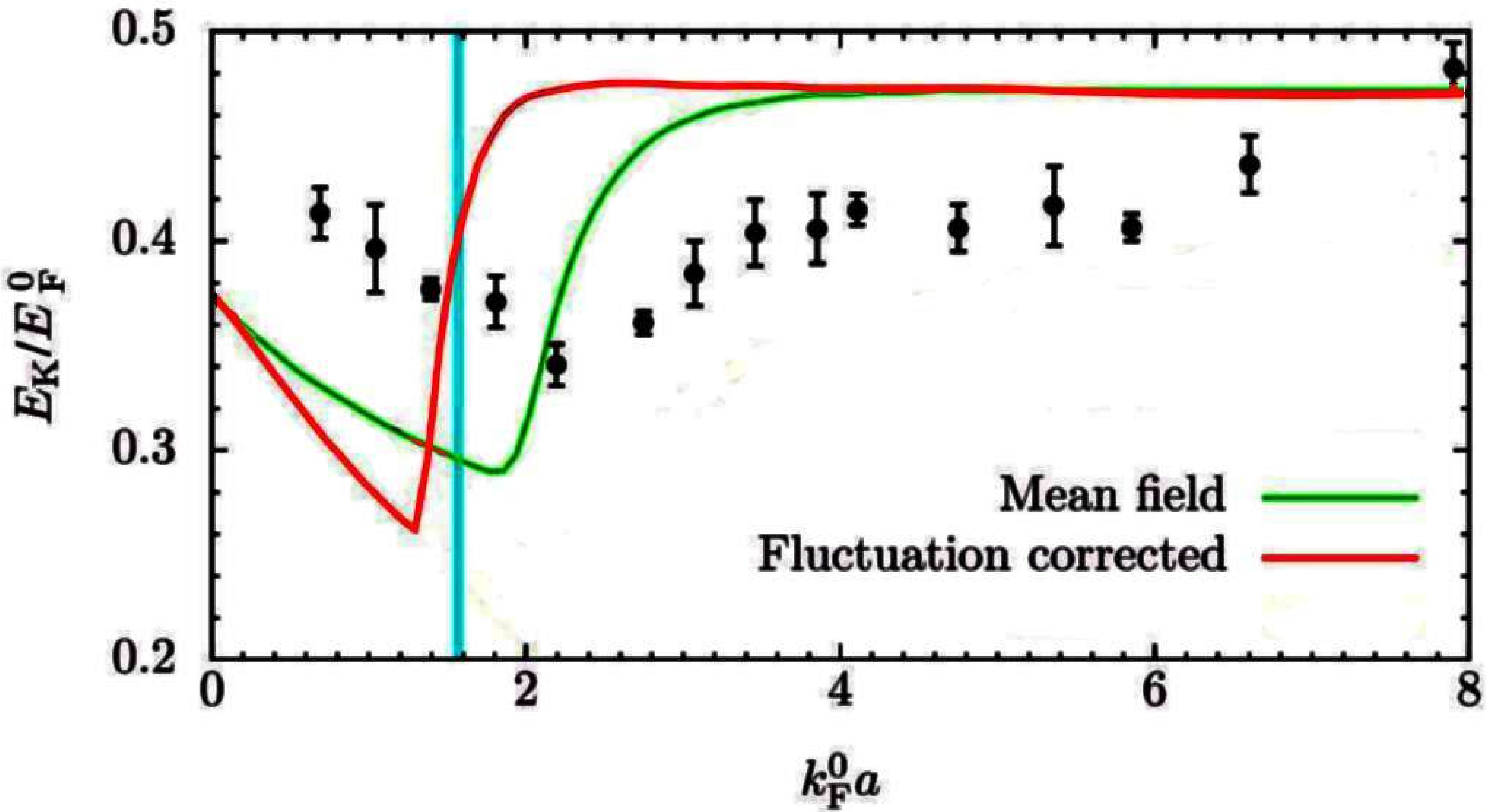
- Verified by *ab initio* Quantum Monte Carlo calculations[2]



[1]Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Fluctuation corrections

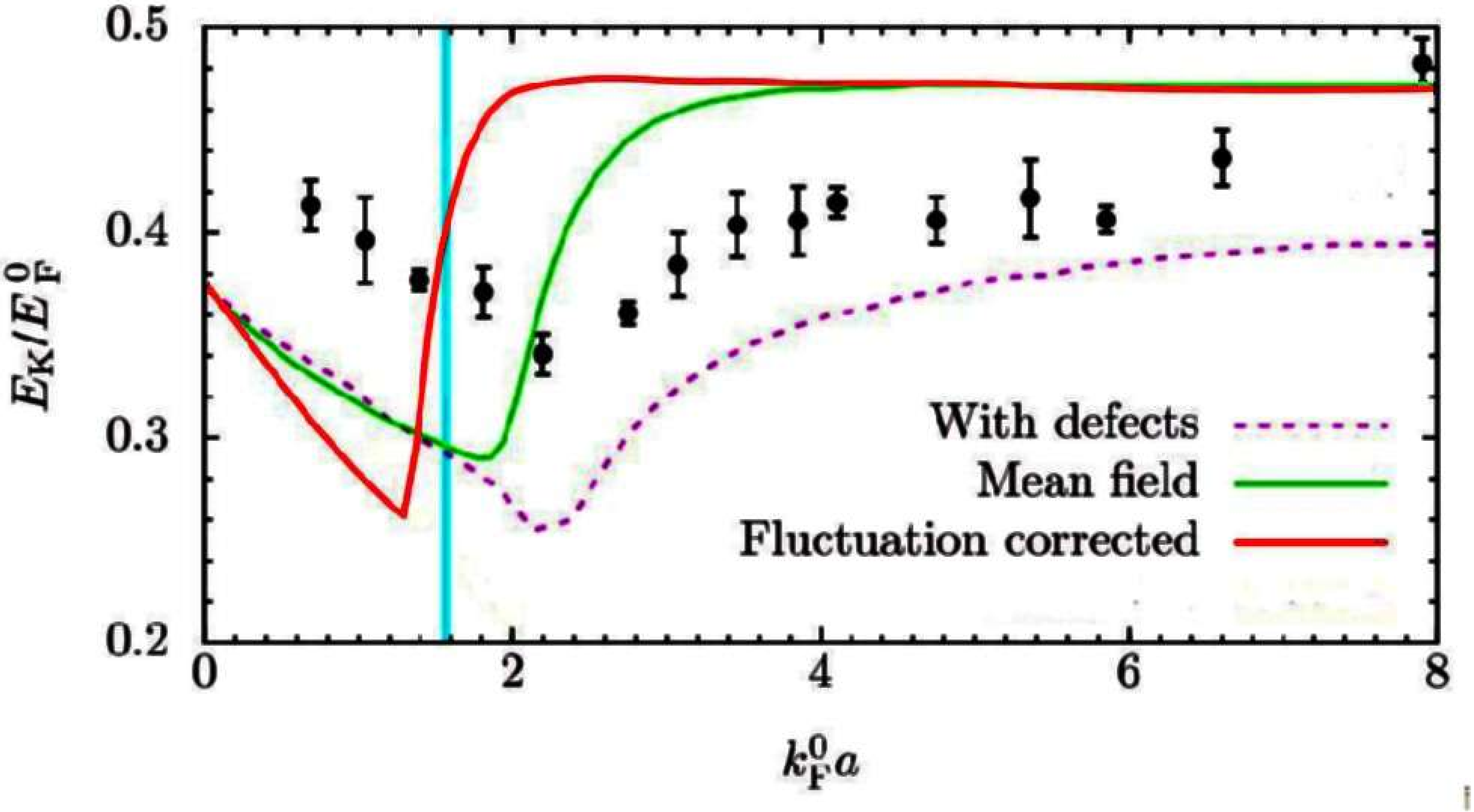
- Extend theory through fluctuation corrections



Conduit & Simons, Phys. Rev. A **79**, 053606 (2009) &
Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Consequences of atom loss

- Three body atom loss rate $\lambda'[n_\uparrow(\mathbf{r}) + n_\downarrow(\mathbf{r})]n_\uparrow(\mathbf{r})n_\downarrow(\mathbf{r})$ forces experiment to be performed out-of-equilibrium



Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

Damping of fluctuations by atom loss

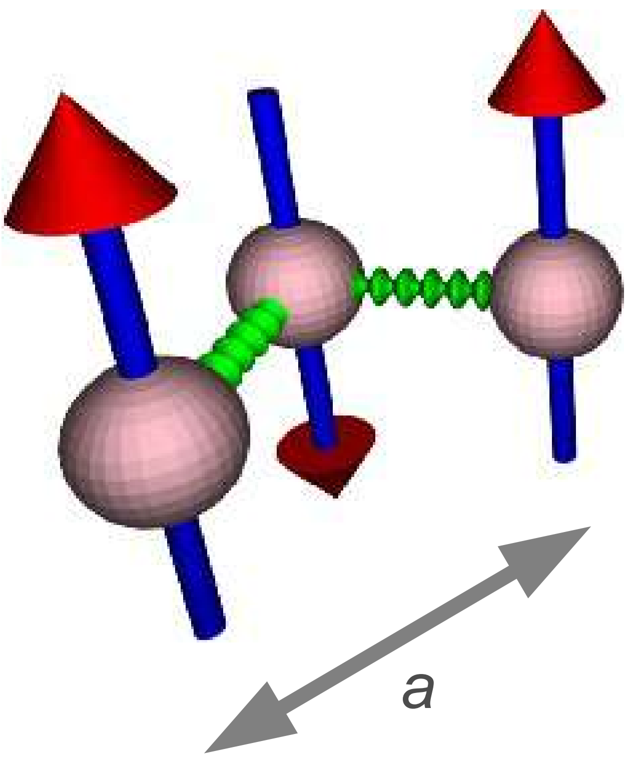
- Atom loss rate is

  $\lambda' \chi(\mathbf{r}-\mathbf{r}')[c_\uparrow^\dagger(\mathbf{r}')c_\uparrow(\mathbf{r}') + c_\downarrow^\dagger(\mathbf{r}')c_\downarrow(\mathbf{r}')]c_\uparrow^\dagger(\mathbf{r})c_\downarrow^\dagger(\mathbf{r})c_\downarrow(\mathbf{r})c_\uparrow(\mathbf{r})$

- A mean-field approximation, $\overline{N} = n_\uparrow(\mathbf{r}') + n_\downarrow(\mathbf{r}')$ places interactions on same footing as interactions

  $S_{int} = (g + i\lambda\overline{N})c_\uparrow^\dagger(\mathbf{r})c_\downarrow^\dagger(\mathbf{r})c_\downarrow(\mathbf{r})c_\uparrow(\mathbf{r})$

- Also include atom source $-i\gamma c_\sigma^\dagger c_\sigma$ to ensure gas remains at equilibrium

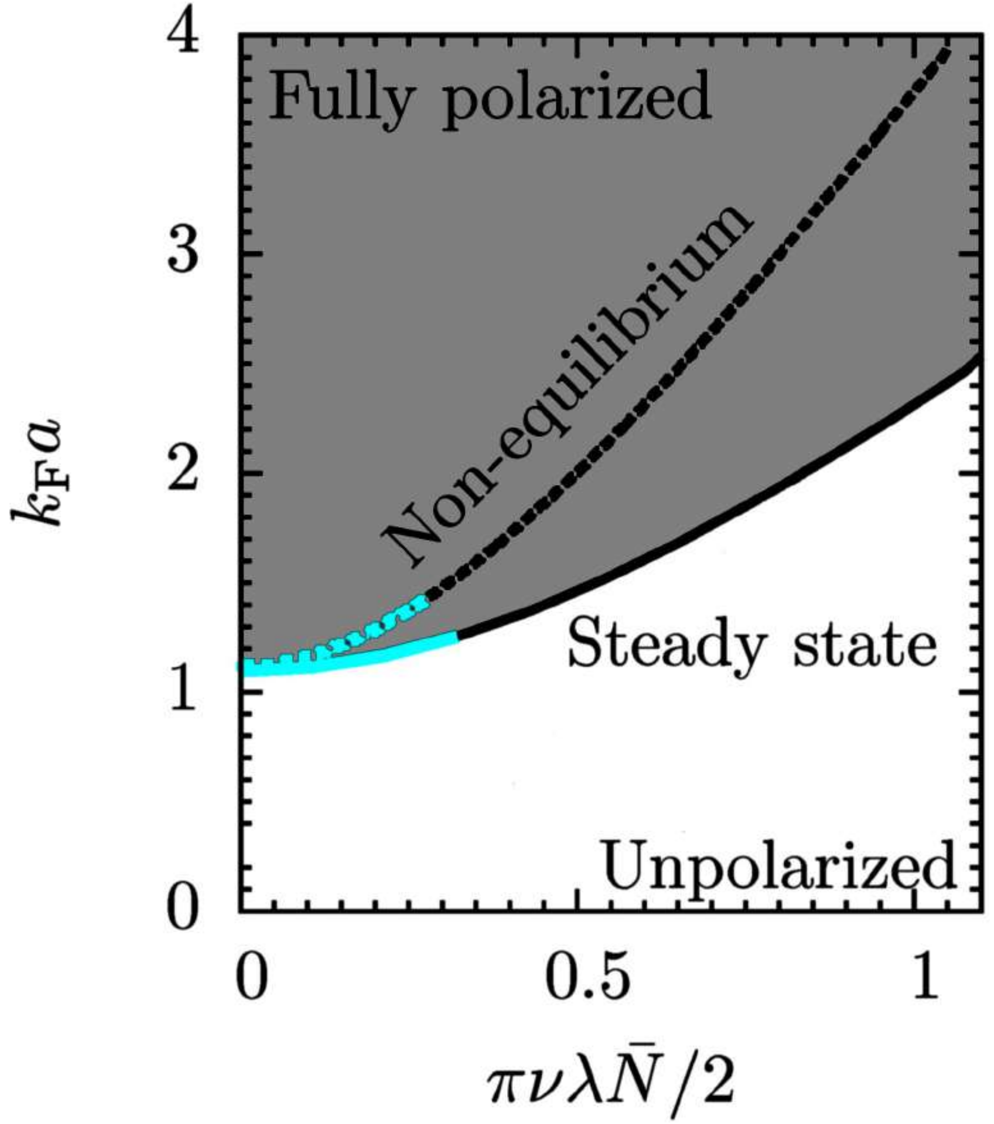


- Loss damps fluctuations so inhibits the transition

$$F = F_0 + \frac{1 - g\nu}{2\nu} m^2 + u m^4 + v m^6 + \left(g^2 - \lambda^2 \overline{N}^2\right)\left(r m^2 + w m^4 \ln|m|\right)$$

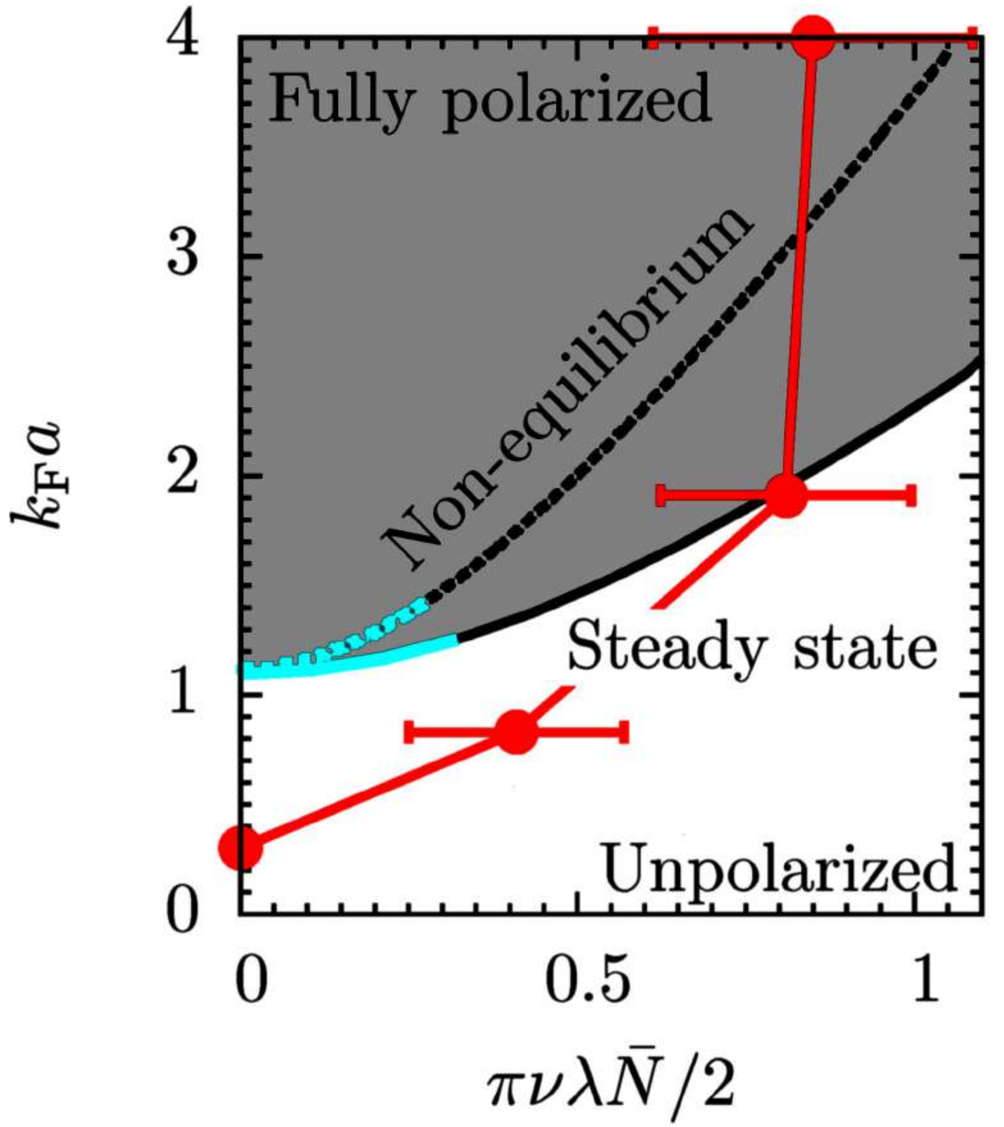
Phase boundary with atom loss

- Atom loss raises the interaction strength required for ferromagnetism



Conduit & Altman, arXiv: 0911.2839

Interaction renormalization with atom loss



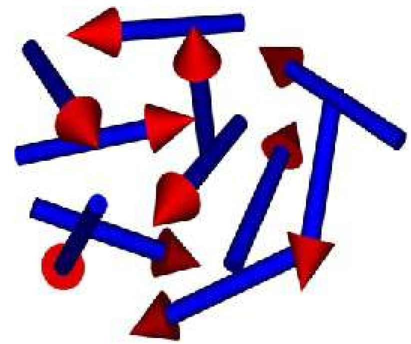
Conduit & Altman, arXiv: 0911.2839; Huckans *et al*. PRL **102**, 165302 (2009)

Summary

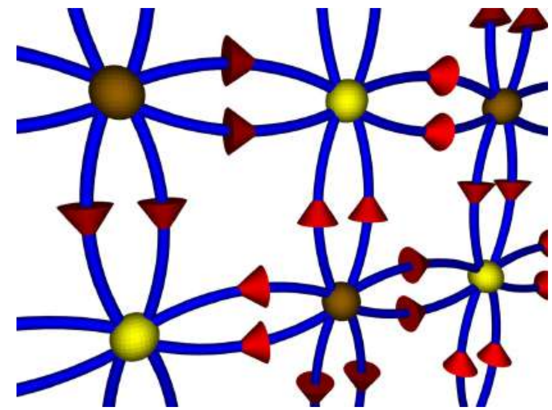
- Mean-field theory provides a reasonable qualitative description of the transition
- Discrepancy in the interaction strength could be accounted for by:

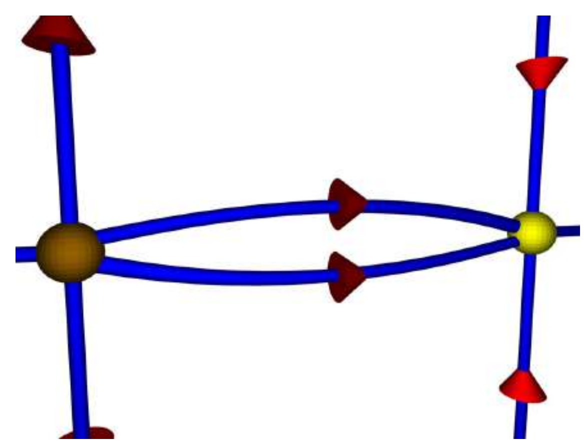
1) Non-equilibrium formation of the ferromagnetic phase
2) Renormalization of interaction strength due to atom loss

Condensation of topological defects



Start in paramagnetic state and ramp up interaction strength



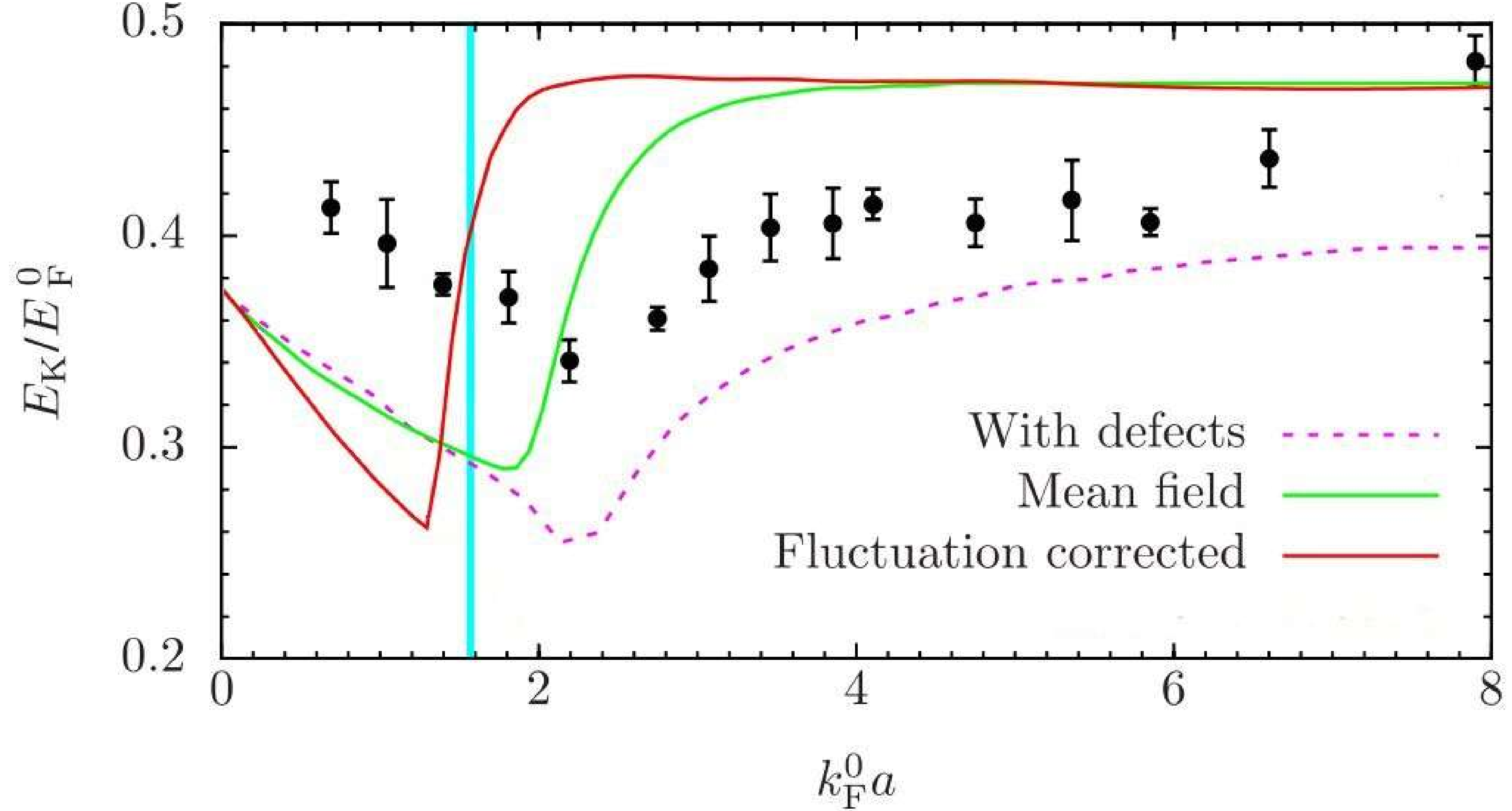


Mutual annihilation of defects

- Defects freeze out from disordered state
- Defect annihilation hinders the formation of the ferromagnetic phase thus raising the required interaction strength
- Defect radius $L \sim t^{½}$ [Bray, Adv. Phys. **43**, 357 (1994)]

Condensation of topological defects

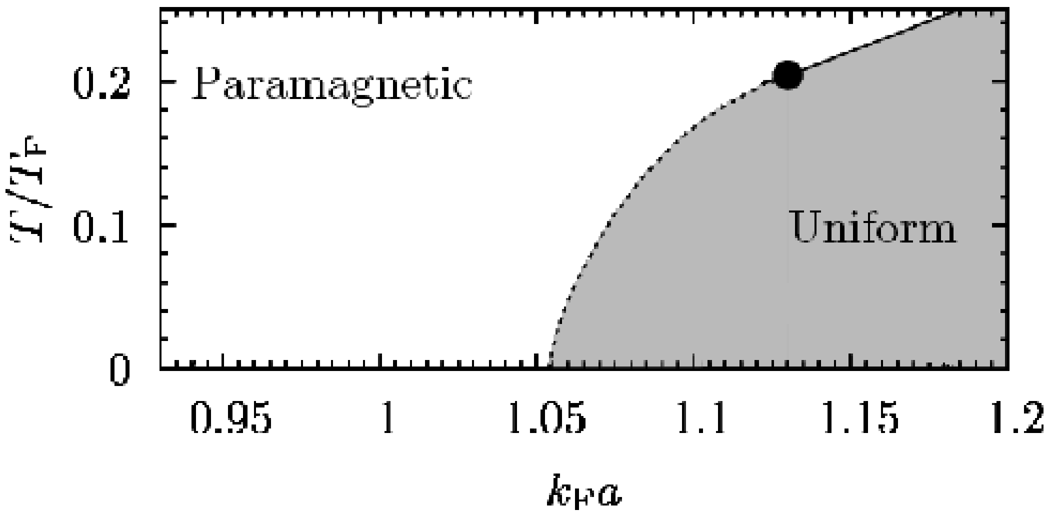
- Condensation of defects inhibits the transition



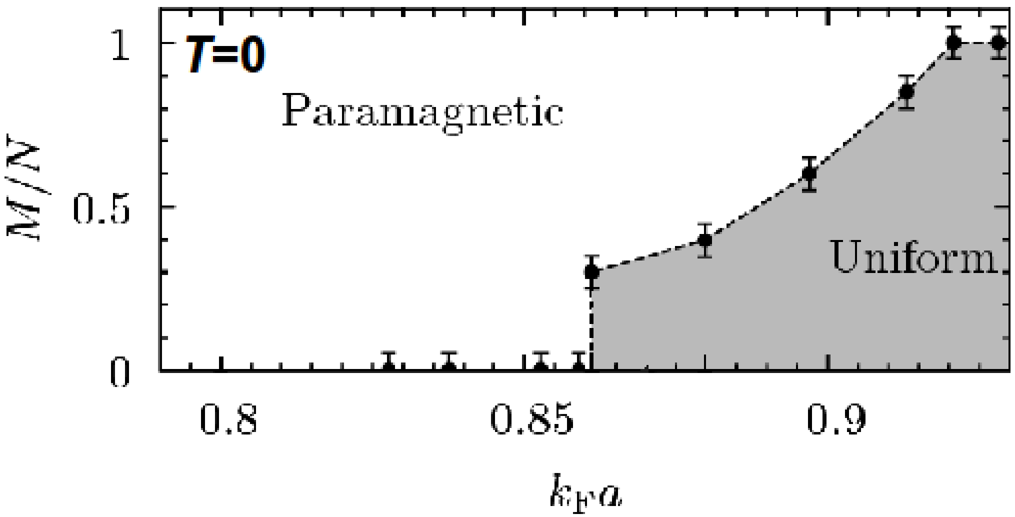
Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

First order phase transition and Quantum Monte Carlo verification

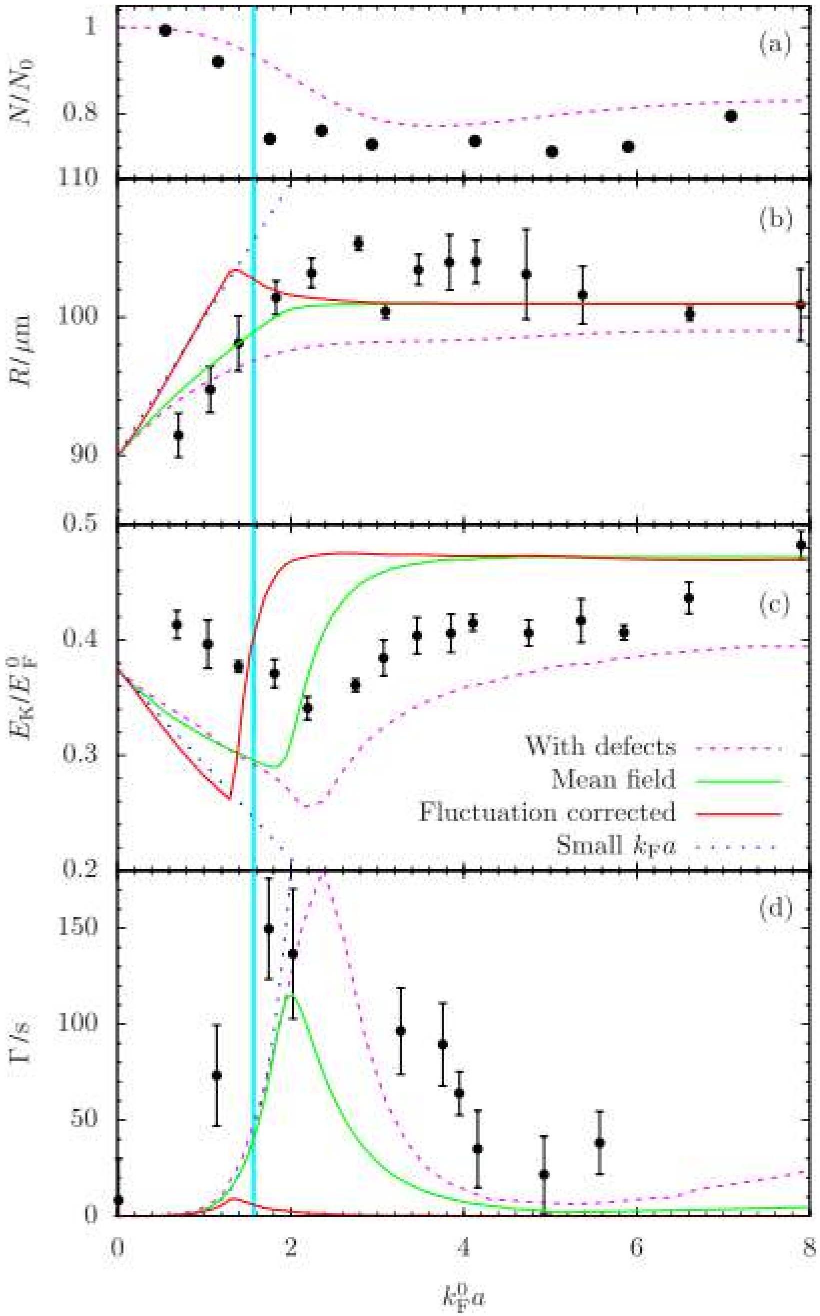
- First order transition into uniform phase with TCP



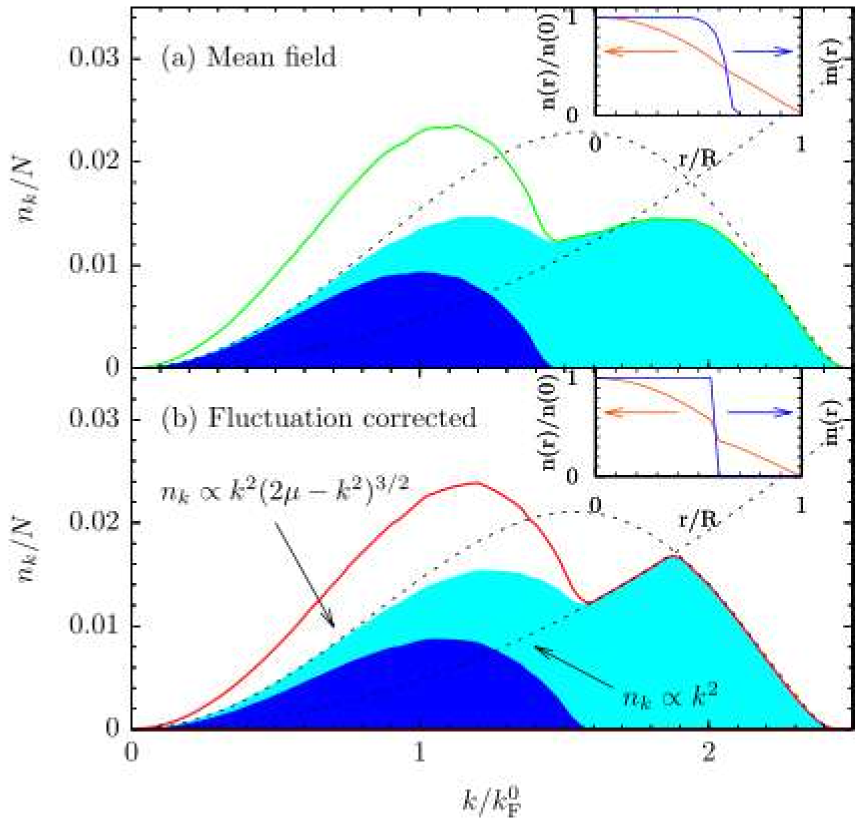
- QMC also sees first order transition



Summary of equilibrium results



Momentum distribution



New approach to fluctuation corrections

$$Z = \int D\psi \exp\left(-\int \sum_{\sigma} \bar{\psi}_{\sigma}(-i\omega+\epsilon-\mu)\psi_{\sigma} - g\int \bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Analytic strategy:

  1) Decouple in both the density and spin channels (previous approaches employ only spin)

  2) Integrate out electrons

  3) Expand about uniform magnetisation

  4) Expand density and magnetisation fluctuations to second order

  5) Integrate out density and magnetisation fluctuations
- Aim to uncover connection to second order perturbation theory
- Unravel origin of logarithmic divergence in energy and relation to tricritical point structure

Analytical method

- System free energy $F=-k_B T \ln Z$ is found via the partition function

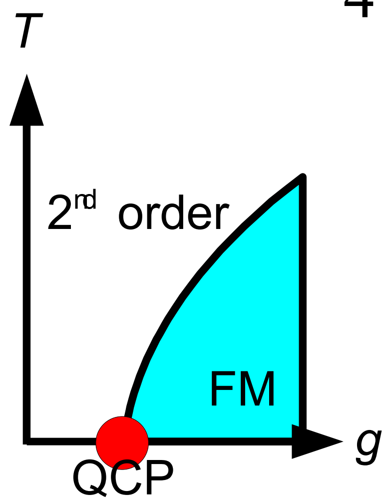
$$Z=\int D\psi \exp\left(-\int \sum_\sigma \bar{\psi}_\sigma(-i\omega+\epsilon-\mu)\psi_\sigma - g\int \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow\right)$$

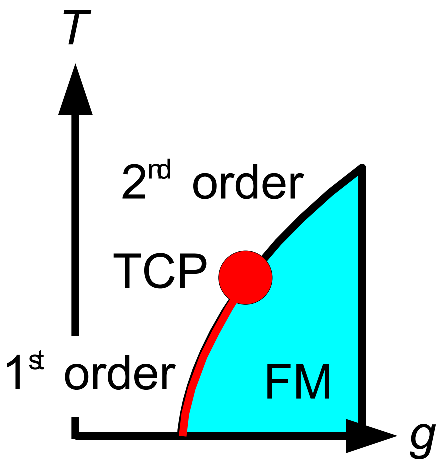
- Decouple using only the average magnetisation $m=\bar{\psi}_\downarrow \psi_\uparrow - \bar{\psi}_\uparrow \psi_\downarrow$

  gives $F \propto (1-g\nu)m^2$ i.e. the Stoner criterion

- Belitz-Kirkpatrick-Vojta (soft particle-hole & magnetisation) [Belitz *et al.* Z. Phys. B 1997]

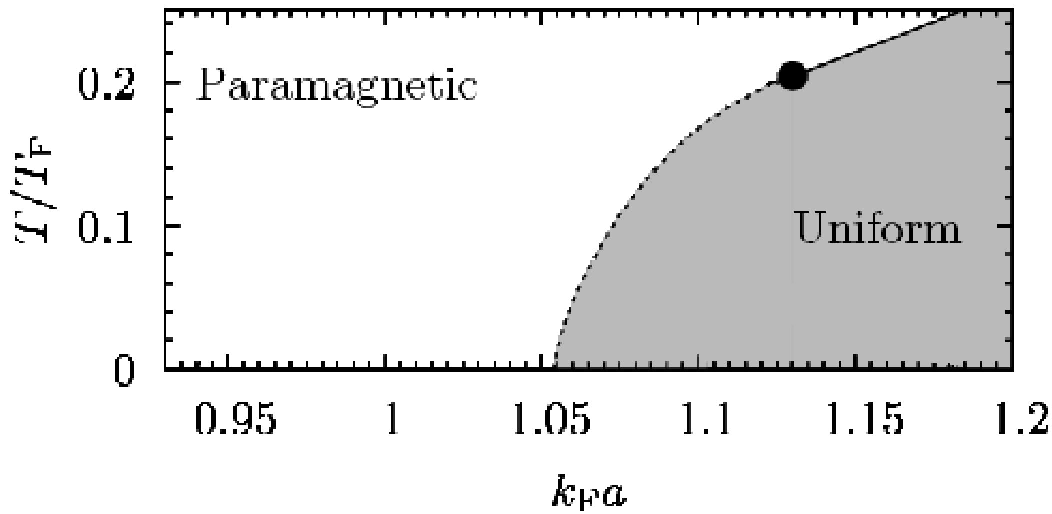
$$F=\frac{1}{2}\left(|\omega|/\Gamma_q+r+q^2\right)m^2+\frac{u}{4}m^4\ln(m^2+T^2)+\cdots-hm$$



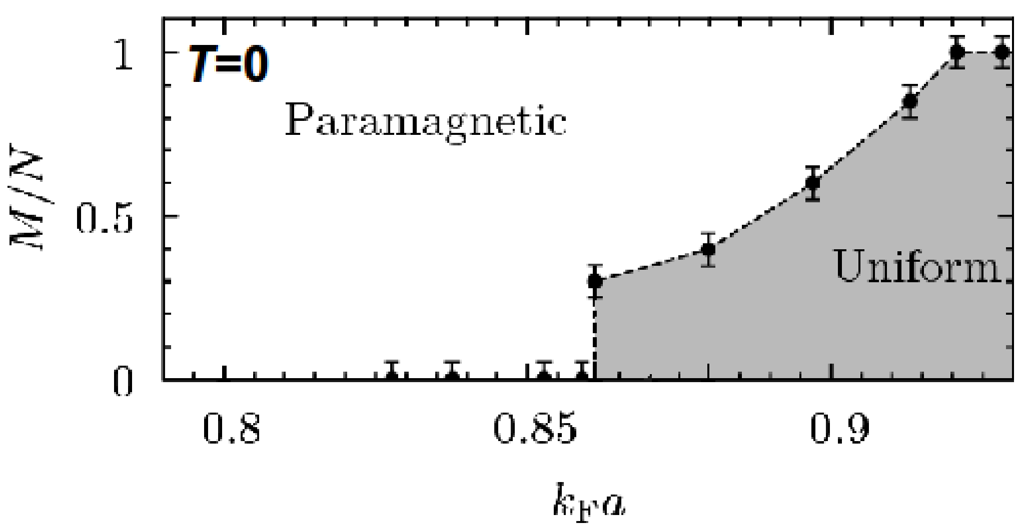


Quantum Monte Carlo verification

- First order transition into uniform phase with TCP



- QMC also sees first order transition



Cold atomic gases — spin

- Two fermionic atom species have a *pseudo-spin*:

^{6}Li	$m_F=1/2$	maps to	spin 1/2
^{6}Li	$m_F=-1/2$	maps to	spin -1/2

- The up-and down spin particles *cannot* interchange — population imbalance is fixed. Possible spin states are:

$\lvert\uparrow\uparrow\rangle$	$S=1$, $S_z=1$	State not possible as S_z has changed
$\lvert\downarrow\downarrow\rangle$	$S=1$, $S_z=-1$	State not possible as S_z has changed
$\left(\lvert\uparrow\downarrow\rangle+\lvert\downarrow\uparrow\rangle\right)/\sqrt{2}$	$S=1$, $S_z=0$	Magnetic moment in plane
$\left(\lvert\uparrow\downarrow\rangle-\lvert\downarrow\uparrow\rangle\right)/\sqrt{2}$	$S=0$, $S_z=0$	Non-magnetic state

- Ferromagnetism, if favourable, must form in-plane

Particle-hole perspective

- To second order in g the free energy is

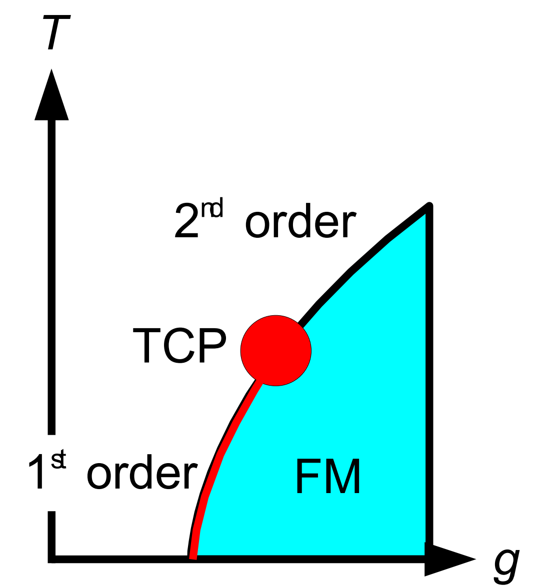
$$F=\sum_{\sigma,\boldsymbol{k}}\epsilon_{\boldsymbol{k}}^{\sigma}n(\epsilon_{\boldsymbol{k}}^{\sigma})+gN^{\uparrow}N^{\downarrow}$$

$$-\frac{2g^2}{V^3}\sum_{\boldsymbol{p}}\int\int\frac{\rho^{\uparrow}(\boldsymbol{p},\epsilon_{\uparrow})\rho^{\downarrow}(-\boldsymbol{p},\epsilon_{\downarrow})}{\epsilon_{\uparrow}+\epsilon_{\downarrow}}d\epsilon_{\uparrow}d\epsilon_{\downarrow}$$

$$+\frac{2g^2}{V^3}\sum_{\boldsymbol{k}_{1,2,3,4}}\frac{n(\epsilon_{\boldsymbol{k}_1}^{\uparrow})n(\epsilon_{\boldsymbol{k}_2}^{\downarrow})}{\epsilon_{\boldsymbol{k}_1}^{\uparrow}+\epsilon_{\boldsymbol{k}_2}^{\downarrow}-\epsilon_{\boldsymbol{k}_3}^{\uparrow}-\epsilon_{\boldsymbol{k}_4}^{\downarrow}}\delta(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k}_3)$$

with $\epsilon_{\boldsymbol{k}}^{\sigma}=\epsilon_{\boldsymbol{k}}+\sigma gm$ and a particle-hole density of states

$$\rho^{\sigma}(\boldsymbol{p},\epsilon)=\sum_{\boldsymbol{k}}n(\epsilon_{\boldsymbol{k}+\boldsymbol{p}/2}^{\sigma})\left[1-n(\epsilon_{\boldsymbol{k}-\boldsymbol{p}/2}^{\sigma})\right]\delta\left(\epsilon-\epsilon_{\boldsymbol{k}+\boldsymbol{p}/2}^{\sigma}+\epsilon_{\boldsymbol{k}-\boldsymbol{p}/2}^{\sigma}\right)$$

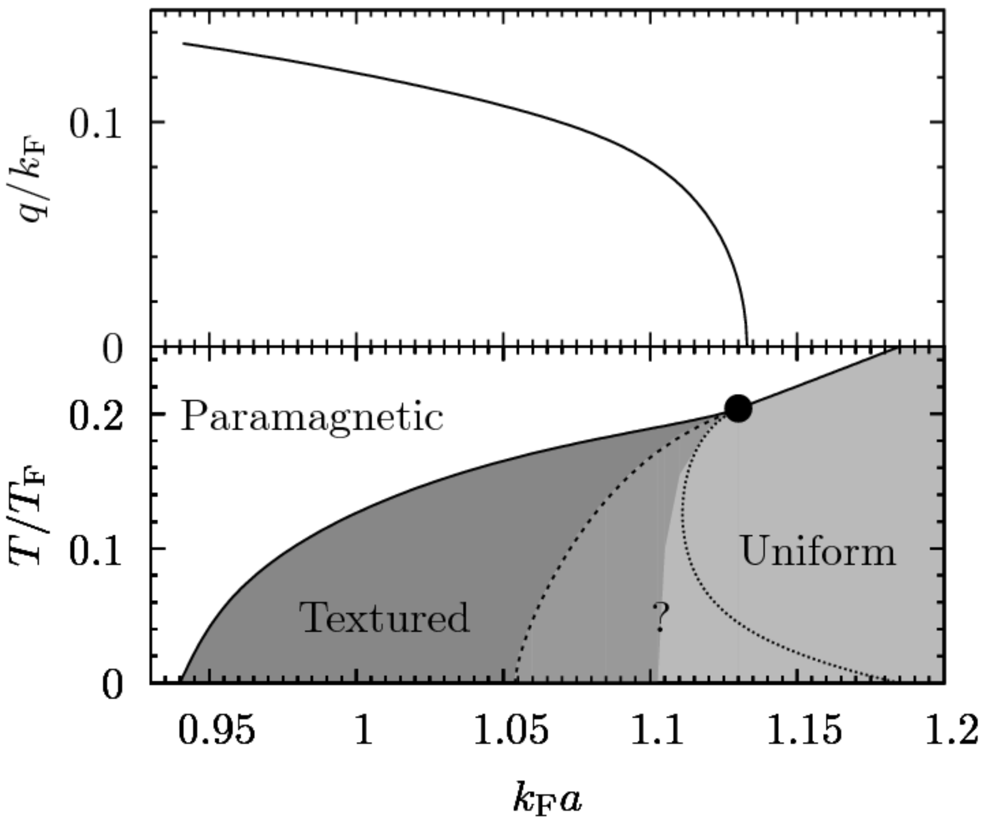


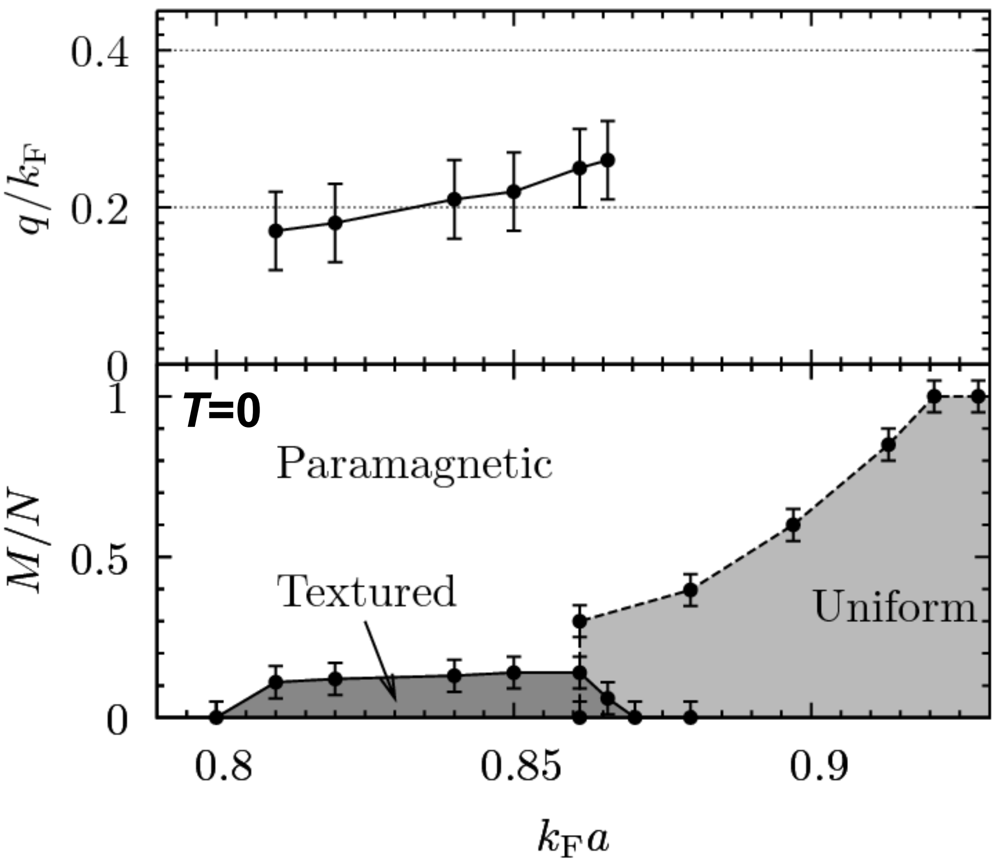
- Enhanced particle-hole phase space at zero magnetisation drives transition first order
- Recover $m^4\ln m^2$ at T=0
- Links quantum fluctuation to second order perturbation approach[1]

[1]Abrikosov 1958 & Duine & MacDonald 2005

Quantum Monte Carlo: Textured phase

- Textured phase preempted transition with $q=0.2k_F$





Modified collective modes

- Collective mode dispersion

$$\Omega = \frac{q^2}{2}\left(1 - \frac{2^{5/3}3}{5k_{\mathrm{F}}a}\frac{1}{1+\tilde{\lambda}^2/(k_{\mathrm{F}}a)^2}\right)$$

- Collective mode damping

$$\Gamma = \frac{q^2}{2}\frac{2^{5/3}3\tilde{\lambda}}{5(k_{\mathrm{F}}a)^2}\frac{1}{1+\tilde{\lambda}^2/(k_{\mathrm{F}}a)^2}$$