LECTURE 1

- Condensed matter physics juxtaposition of interactions and quantum mechanics
- Many body interactions : stars in galaxies
- Quantum mechanics - diffraction
- Combination gives counterintuitive phenomena.

- Real life solids have electrons travelling over lattice
- Basis sets for the electrons
  - Real space
  - Reciprocal space
- latter preferable for conducting freely moving electrons, known as itinerant
- Study physics, descriptions and consequences of interactions in these systems.

Non interacting system:

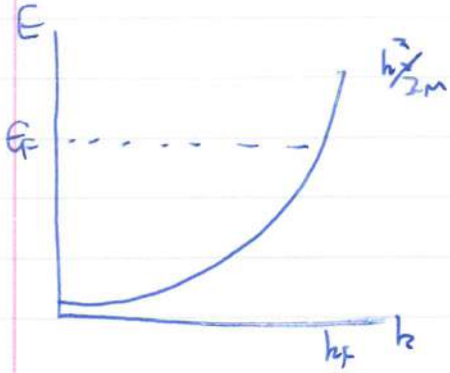
- Hamiltonian $\begin{vmatrix} e^{ik_1 r_{1\uparrow}} & e^{ik_2 r_{1\uparrow}} \\ e^{ik_1 r_{2\uparrow}} & e^{ik_2 r_{2\uparrow}} \end{vmatrix} \begin{vmatrix} e^{-ik_1 r_{1\downarrow}} & e^{-ik_1 r_{2\downarrow}} \\ e^{-ik_2 r_{1\downarrow}} & e^{-ik_2 r_{2\downarrow}} \end{vmatrix} = \begin{vmatrix} \phi(r_{1\uparrow}-r_{1\downarrow}) & \phi(r_{1\uparrow}-r_{2\downarrow}) \\ \phi(r_{2\uparrow}-r_{1\downarrow}) & \phi(r_{2\uparrow}-r_{2\downarrow}) \end{vmatrix}$

- Found in alkali metals, cold atom gases

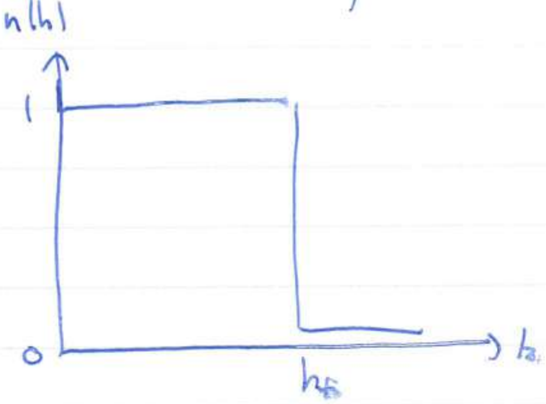
- Representation:
  Slater determinant
  Second quantised

- kinetic energy
$$\frac{\frac{1}{(2\pi)^3}\int \frac{1}{2m} k^2 \cdot 4\pi k^2 \cdot dk}{\frac{1}{(2\pi)^3}\int 4\pi k^2 \cdot dk}$$
$$= \frac{3}{5} E_F$$

- Band structure



Distribution function

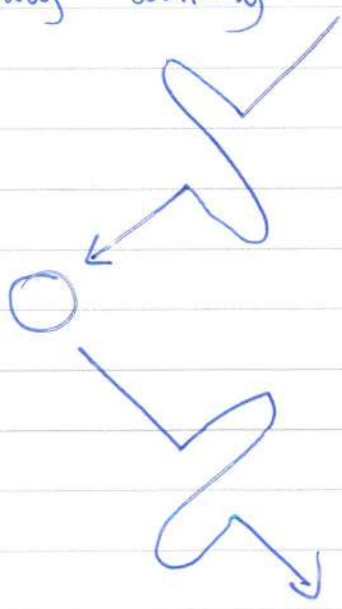


Weak interactions

- Hamiltonian, contact

- Found in screened electrons - heavy fermion materials
    - Opposite spin interacting only as fermions
- Found in cold atom gases - opposite spin only.

Two body scattering:



$$\hat{H} = -\frac{1}{2r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) + \frac{1}{2r^2\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\psi}{d\theta}\right) + \frac{1}{2r^2\sin^2\theta}\frac{d^2\psi}{d\phi^2}$$

with $\psi = u/r$ get

$$\hat{H} = -\frac{1}{2}\frac{d^2u}{dr^2}$$

Non-interacting eigenstate

$$\frac{\sin kr}{r}$$

With interactions

$$\psi = \frac{\sin(kr+\delta)}{r}$$

$$r\psi = \sin(kr+\delta)$$

$$= \sin kr . \cos\delta + \cos kr . \sin\delta$$

$$= \sin\delta\left(\cot\delta \sin kr + \cos kr\right)$$

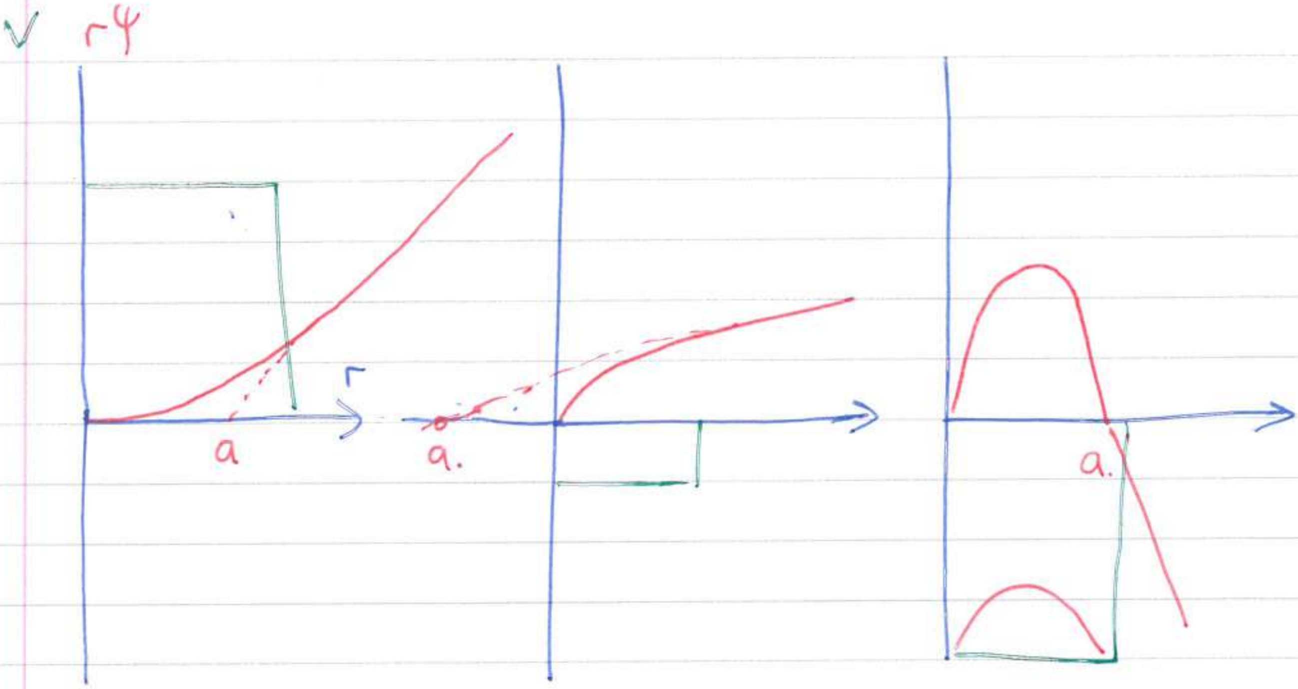
$$\sim \sin\delta\left(\cot\delta . kr + 1\right)$$

for small $k \ll \frac{1}{r}$

look at node; call a

$$\cot\delta = -\frac{1}{ka}$$

Graphical meaning: node in Ψ_r.



Can make effective repulsive interaction with underlying attractive well, but get bound state

To enter bound state need 3-body interaction that has a long relaxation time / Pauli blocked.

Real cold atoms: ground state is lump of solid.

Next order term, effective range:

$$\cot \delta = -\frac{1}{ka} + \frac{1}{2} k R_{eff}$$